Exercises Chapter 2 - Fourier optics and imaging

Exercises from Goodman's book 4th edition:

6-7. A normally incident, unit-amplitude, monochromatic plane wave illuminates a converging lens of 5 cm diameter and 2 meters focal length (see Fig. P6.7). One meter behind the lens and centered on the lens axis is placed an object with amplitude transmittance

$$t_A(\xi,\eta) = \frac{1}{2}\left[1+\cos(2\pi f_o \xi)\right] \operatorname{rect}\left(\frac{\xi}{L}\right) \operatorname{rect}\left(\frac{\eta}{L}\right).$$

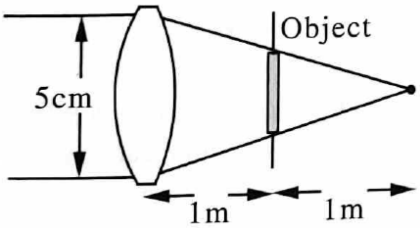
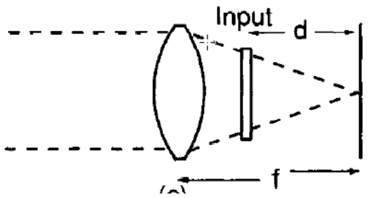


Figure P6.7

Assuming $L = 1$ cm, $\lambda = 0.633\ \mu$m, and $f_o = 10$ cycles/mm, sketch the intensity distribution across the u axis of the focal plane, labeling the numerical values of the distance between diffracted components and the width (between first zeros) of the individual components.

Hint: Note that this case



is mathematically the same as having a lens right at the plane of the object but with a focal length of *d.* This case is treated in Section 6.2.3 of Goodman's.

6-8. In Fig. P6.8 a monochromatic point source is placed a fixed distance z_1 to the left of a positive lens (focal length *f*), and a transparent object is placed a variable distance *d* to the left of the lens. The distance z_1 is greater than *f*. The Fourier transform and the image of the object appear to the right of the lens.

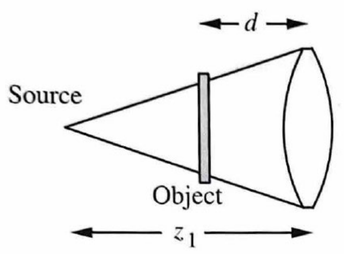


Figure P6.8

(a) How large should the distance *d* be (in terms of z_1 and *f*) to ensure that the Fourier plane and the object are equidistant from the lens?
(b) When the object has the distance found in part (a) above, how far to the right of the lens is its image and what is the magnification of that image?

7-1. The mask shown in Fig. P7.1 is inserted in the exit pupil of an imaging system. Light from the small openings interferes to form a fringe in the image plane.

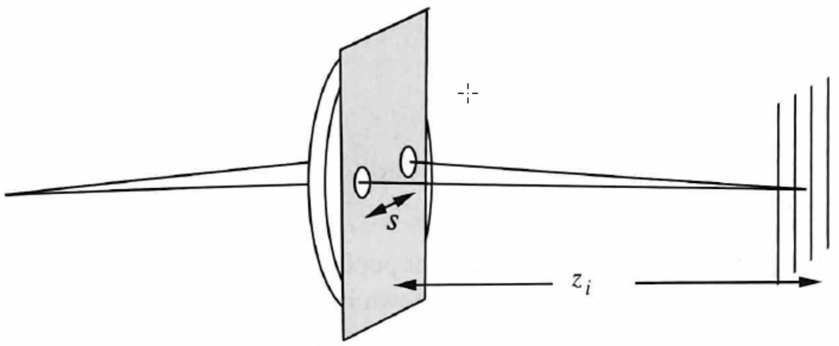


Figure P7.1

(a) Find the spatial frequency of this fringe in terms of the center-to-center spacing s of the two openings, the wavelength λ, and the image distance z_i.

(b) The openings are circular with diameter d. Specify the envelope of the fringe pattern caused by the finite openings in the pupil plane.

You can skip 7-2(c) as we did not cover step-response

7-2. The *line-spread function* of a two-dimensional imaging system is defined to be the response of that system to a one-dimensional delta function passing through the origin of the input plane.

(a) In the case of a line excitation lying along the x axis, show that the line-spread function l and the point-spread function p are related by

$$l(y) = \int_{-\infty}^{\infty} p(x, y)\, dx,$$

where l and p are to be interpreted as amplitudes or intensities, depending on whether the system is coherent or incoherent, respectively.

(b) Show that for a line source oriented along the x axis, the (one-dimensional) Fourier transform of the line-spread function is equal to a slice through the (two-dimensional) Fourier transform of the point-spread function, the slice being along the f_Y axis. In other words, if $\mathcal{F}\{l\} = L$ and $\mathcal{F}\{p\} = P$, then $L(f) = P(0, f)$.

(c) Find the relationship between the line-spread function and the step response of the system, i.e. the response to a unit step excitation oriented parallel to the x axis.

7-7. Consider a *pinhole camera* shown in Fig. P7.7.
Assume that the object is incoherent and nearly monochromatic, the distance z_o from the object is so large that it can be treated as infinite, and the pinhole is circular with diameter w.

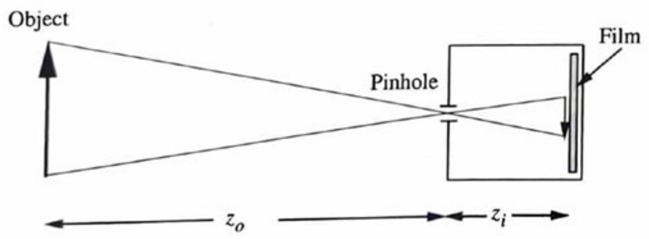


Figure P7.7

(a) Under the assumption that the pinhole is large enough to allow a purely geometrical-optics estimation of the point-spread function, find the optical transfer function of this camera. If we define the "cutoff frequency" of the camera to be the frequency where the first zero of the OTF occurs, what is the cutoff frequency under the above geometrical-optics approximation? (Hint: First find the intensity point-spread function, then Fourier transform it. Remember the second approximation above.)

(b) Again calculate the cutoff frequency, but this time assuming that the pinhole is so small that Fraunhofer diffraction by the pinhole governs the shape of the point-spread function.

(c) Considering the two expressions for the cutoff frequency that you have found, can you estimate the "optimum" size of the pinhole in terms of the various parameters of the system? Optimum in this case means the size that produces the highest possible cutoff frequency.

Hint: Under very severe defous as in (a) the PSF becomes a geometrically magnified, or demagnified, image of the pupil.

How does this compare to the pinhole resolution equation

$$d = \sqrt{2.44}\sqrt{f\lambda} = 1.562\sqrt{f\lambda}$$

7-10. An object with a square-wave amplitude transmittance (shown in Fig. P7.10) is imaged by a lens with a circular pupil function. The focal length of the lens is 10 cm, the fundamental frequency of the square wave is 100 cycles/mm, the object distance is 20 cm, and the wavelength is 1 μm. What is the minimum lens diameter that will yield *any variations* of intensity across the image plane for the cases of

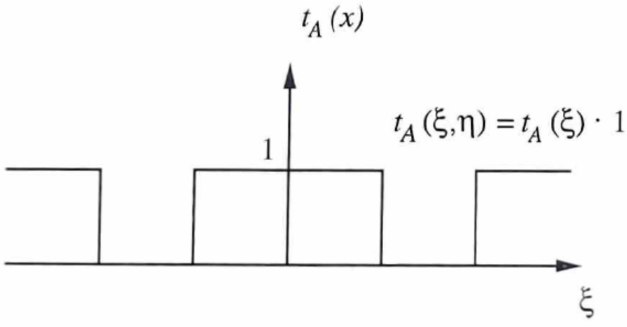


Figure P7.10

(a) Coherent object illumination?

(b) Incoherent object illumination?

7-13. The *F-number* of a lens with a circular aperture is defined as the ratio of the focal length to the lens diameter. Show that when the object distance is infinite, the cutoff frequency for a coherent imaging system using this lens is given by $f_o = \frac{1}{2\lambda F\#}$, where $F\#$ represents the F-number.